General Slowing or Decreased Inhibition? Mathematical Models of Age Differences in Cognitive Functioning

Sy-Miin Chow and John R. Nesselroade

Department of Psychology, University of Virginia, Charlottesville.

Researchers have attempted to explain age-related decrements in cognitive performance in terms of reduced processing speed or decreased ability to inhibit irrelevant thoughts. We present these ideas in the context of a dynamic model derived from extensions of the classical predator–prey equation. Reduced processing speed among older adults is represented by use of delays in the dynamic model, whereas the interference imposed by distractors is captured by use of the predator–prey interaction term. We demonstrate the versatility of this modeling approach, and its pertinence to age-related behavioral change, by means of numerical simulations. In showing the applicability of these models, we identify several unresolved methodological and measurement issues that have to be addressed.

For the past several decades, there has been considerable interest in explaining age-related declines in cognitive performance. Currently, some of the most frequently endorsed theoretical viewpoints assume reduced processing speed in older adults (e.g., Salthouse, 1996b) and deficits in older adults' inhibitory mechanisms (Hasher & Zacks, 1988) to be key contributors to age-related decline. On one hand, with the processing-speed theory, Salthouse (1996b) proposed that reduced processing speed in older adults leads to slower execution of cognitive operations as well as the loss of information from early processing, thus impairing older adults' cognitive performance. Hasher and Zacks (1988), on the other hand, argued that older adults' cognitive impairments spring from their decreased ability to inhibit irrelevant information.

Reduced processing speed is generally recognized as one of the common consequences of aging (Kane, Hasher, Stoltzfus, Zacks, & Connelly, 1994). Evidence suggesting the possibility of decreased inhibition, however, is mixed. Some studies have revealed a facilitatory effect in older adults' responses just as in younger adults' responses (i.e., they were faster in rejecting a distractor that had appeared in previous trials), but not negative priming (i.e., the slowing in responses when a previous distractor becomes a target)—a finding that supports Hasher and Zack's (1988) argument (see, e.g., McDowd & Oseas-Kreger, 1991; Hedden & Park, 2001). Others have found that the magnitudes of negative priming exhibited by older adults were just about the same, if not greater, than those observed among younger adults (see, e.g., Langley, Overmier, Knopman, & Prod'Homme, 1998; Little & Hartley, 2000). In still other studies, the speculated age differences in susceptibility to interference did not even emerge (e.g., Myerson, Hale, Rhee, & Jenkins, 1999).

Although some researchers do acknowledge the influence of both decreased inhibition and general slowing on older adults' performance (e.g., Kane et al., 1994), the mechanisms by which they operate or interact are less clearly understood. Salthouse and Meinz (1993), for instance, observed that measures of interference did not account for unique age-related variance over and above that accounted for by processing speed. Given the shared age-related differences observed in a wide range of cognitive tasks, Salthouse (1996a) argued that a small number of common factors, rather than a large number of unique factors, might be responsible for inducing age-related differences in cognitive performance. The former argument might well be a more parsimonious view than the latter, but the dynamics underlying changes in performance levels can be an altogether different research question.

The dynamics of a system are what we seek to capture by using the models proposed in this study. Patterns of individual differences in levels of performance might be similar across a wide range of cognitive tasks (thus yielding one general "factor"), but the determinants of these patterns and their relationships over time cannot be adequately explained by factor analysis alone. With recent advances in dynamical systems analyses, more modeling tools are now available to capture the dynamics underlying self-regulatory cognitive processes. (The word "self-regulatory" is used to describe a system that evolves according to its own intrinsic dynamics, or a process that is affected by its own level, and hence self-regulates over time. In self-regulation, an individual undergoes an intrinsic process of change either with or without the presence of external perturbations.) For instance, neural network models that capitalize heavily on the notion of parallel distributed processing (Rumelhart, McClelland, & PDP, 1986) are examples of self-regulatory dynamical systems; (see, e.g., the overhead model of Cerella [1990], the information loss model of Myerson, Hale, Wagstaff, Poon, and Smith [1990], and the molar entropy model of Allen, Kaufman, Smith, and Propper [1998a, 1998b]). In fact, the idea of a higher entropy level, or internal noise, in older than younger adults as suggested by the model of Allen and colleagues (1998a, 1998b) has some close parallels to the notion of decreased inhibition proposed by Hasher and Zacks (1988).

As an alternative to neural network models, we use differential equations to model the dynamics of age differences in cognitive functioning. Although differential equations have
a substantial history in social sciences (see e.g. Coleman, 1968), they have not gained the momentum in recent years of neural network models. This is due in part to some of the difficulties in fitting differential equations models (particularly nonlinear ones) to empirical data. Despite these methodological constraints, differential equations have clear conceptual appeal in representing the dynamics of a construct. Here we present a series of differential equation models derived from extensions of a predator–prey model, and we demonstrate that the arguments surrounding the general slowing hypothesis and inhibitory deficits view can be reconciled in a rather simple mathematical model. Our goal here is not to discount experimental procedures designed to evaluate these two theories separately; rather, we seek to present an alternative approach designed to evaluate these two theories within a dynamic modeling framework. Before proceeding to details regarding this model, we first present an overview of differential equation models to highlight some of their useful theoretical properties.

Dynamical Models and the Incorporation of Time Delay

We feature differential equations that model an individual’s changes in cognitive performance over time. These changes are, by the nature of differential equations, considered to unfold continuously over time. Differential equations thus emphasize the continuity of the system in the time domain, or, in other words, that the system undergoes a certain “process” (for examples of differential equation modeling, see, e.g., Allen et al., 1998a, 1998b; Thelen & Smith, 1994). This is in contrast to using discrete occasion sampling as a basis for capturing the underlying change processes.

Most mathematical models of predator–prey relations were derived largely from the work of Lotka (1925) and Volterra (cf. Pearce, 1970) and involve the explicit use of ordinary differential equations (ODEs). ODEs are limited in representing the dynamics of a system, however, because they do not provide for time delays, a feature increasingly recognized as a key element of a system’s dynamics (see e.g., Baker, Bocharov, & Rihan, 1999). Incorporating delays, for instance, can produce periodic oscillations or instabilities in the system (Baker et al., 1999). Short delays can also stabilize an otherwise unstable dynamical system (Mackey & Glass, 1977; Nisbet & Gurney, 1982). Newer delay differential equations (DDEs) explicitly include time delays as part of a system’s intrinsic dynamics.

We extend the classical Lotka–Volterra model of predator–prey relations proposed by Chen, Lu, and Wang (1995) to provide a dynamic representation of the nature of age differences in cognitive functioning. The extension involves the use of DDEs to capture features of the dynamics. This enables us to introduce into the models, for example, short processing delays that can lead to temporary perturbations in the responses of older adults, thus simulating what some researchers have called “increased internal noise” (Allen et al., 1998a, 1998b).

In the next section, we present more details of a set of DDEs, and we show how the phenomena of general slowing and decreased inhibition can be reconciled by using this model. This discussion is followed by results from a simulation showing the effects of reduced processing speed and increased interference. Finally, we also outline some of the issues that have to be addressed before nonlinear dynamical models such as the ones proposed in this study can be more readily fitted to empirical data.

MODEL DESCRIPTIONS

Before we extend the predator–prey model to age-related behavior, we first briefly summarize the classical Lotka–Volterra equation in the context of predator–prey relations and examine some variations that have been proposed over the past few decades. The classical Lotka–Volterra model is a set of equations generally written as

\[ \dot{x}_1(t) = x_1(t)[r_1 + a_{12}x_2(t)], \]
\[ \dot{x}_2(t) = x_2(t)[r_2 + a_{21}x_1(t)], \]

where \( x_i(t) \) represents the density of species \( i \) at time \( t \), \( \dot{x}_i(t) \) represents the rate of change in density of species \( i \) at time \( t \), \( r_i \) is the growth rate of species \( i \), and \( a_{ij} \) (\( i, j = 1, 2 \) in this example) represents the interspecies interaction parameter. In the case of predator–prey relations where species \( i \) is the prey and \( j \) is the predator, \( a_{ij} \) will usually be negative and \( a_{ji} \) positive. This is because the density of the predator tends to increase as a result of the interaction whereas the reverse is observed in the prey population. The left-hand side of the two equations, \( \dot{x}_i(t) \), thus represents the rate of change in the density of species \( i \).

In the case in which the interactions between the two systems are competitive in nature, both \( a_{ij} \) and \( a_{ji} \) are negative. If the relationships are cooperative in nature, both \( a_{ij} \) and \( a_{ji} \) will be positive, as the interaction between the two systems leads to mutual benefits.

As noted by authors such as Pearce (1970), the classical Lotka–Volterra model was sometimes criticized as an unrealistic representation of population dynamics. In particular, the model’s prediction of an ongoing oscillation in the predator and prey populations, with the two dropping to zero density in alternating order, may not be feasible in empirical settings. Many researchers have since proposed variations of the original Lotka–Volterra model. When a quadratic term is added to Equations 1 and 2, for example, an intraspecies competition effect can be incorporated into the two systems’ intrinsic dynamics (Pearce, 1970). The quadratic term can be added by having

\[ \dot{x}_1(t) = x_1(t)[r_1 + a_{12}x_2(t) + a_{13}x_1(t)], \]
\[ \dot{x}_2(t) = x_2(t)[r_2 + a_{21}x_1(t) + a_{23}x_2(t)], \]

where \( a_{ii} \) and \( a_{ij} \) < 0, and \( i \) and \( j \) still represent prey and predator, respectively. The \( a_{ii} \) and \( a_{ij} \) parameters capture the magnitude of intraspecies competition in the two populations of predator and prey. If the interspecies interaction terms \( (a_{ij} \) and \( a_{ji} \) are dropped from Equations 3 and 4, they reduce to

\[ \dot{x}_1(t) = x_1(t)[r_1 + a_{12}x_2(t)], \]
\[ \dot{x}_2(t) = x_2(t)[r_2 + a_{23}x_2(t)], \]

which are identical to the familiar logistic equation (May, 1974) involving two, noninteracting species, usually represented in the framework of difference equations (also known as the Verhulst equation in continuous form; cf. Kaplan & Glass, 1995). Van Geert (1991), for example, used the logistic model as a starting point for building other more elaborate models in
the context of human cognitive and language development. The quadratic terms in Equations 5 and 6 thus constitute competition stemming from the same species and impose a limit on the species’ growth capacity.

Finally, if there is a delay in the interspecies and intraspecies interaction processes such that the consequences brought by these interactions only surface after a certain time lag, this delay can be incorporated into Equations 3 and 4. This was a model considered by Chen and colleagues (1995). It is written as

\[ x_1(t) = x_1(0) + a_{11}x_1(t) + a_{12}x_1(t - \tau_{12}) + a_{13}x_1(t - \tau_{13}), \quad (7) \]

\[ x_2(t) = x_2(0) + a_{21}x_2(t) + a_{22}x_2(t - \tau_{22}) + a_{23}x_2(t - \tau_{23}), \quad (8) \]

where \( \tau_{ij} \) and \( \tau_{kl} \) represent the delays in interspecies interactions, whereas \( \tau_{ij} \) and \( \tau_{kl} \) represent the delays in intraspecies interactions. Generally, the delay terms do not have to be the same: \( \tau_{ij} \neq \tau_{kl} \neq \tau_{ml} \). As mentioned earlier, incorporation of these time delays can alter the original dynamics of the systems in critical ways. In the next section, we apply Equations 7 and 8 to cognitive behavior and illustrate how the parameters explicated herein can be linked to age-related cognitive changes.

**Applications to Age-Related Cognitive Change**

If \( x_1(t) \) and \( x_2(t) \) are viewed as task scores (levels) on a primary task and a secondary task, respectively, rather than as densities of two species, the interaction between these two terms can thus represent an interference effect imposed by the secondary task on the primary task. The nature of this interaction is determined by the sign of the interaction parameters, \( a_{ij} \) and \( a_{ji} \) (i.e., whether \( a_{ij} > 0, < 0 \)) If the two tasks compete for limited processing resources, both \( a_{ij} \) and \( a_{ji} \) will be negative. These interaction parameters may assume the conventional predator–prey relations (i.e., \( a_{ij} < 0 \) and \( a_{ji} > 0 \)) if, for example, a distractor, or some otherwise irrelevant information, can “benefit” from interaction with a primary task. For instance, irrelevant information can interfere with an individual’s learning on the primary task by consuming some of the processing capacity otherwise allocated to the primary task, and possibly continue to grow in magnitude in the process (e.g., a small thought cascading into a whole episode of mind wandering).

In actual data, the magnitudes of these parameters provide an estimate of the extent to which an individual is inhibiting interference from another source. The magnitude of this inhibition is often assessed in experimental settings by some form of differencing, for example, how much more time an individual spends on a certain type of stimuli when it is compared with a baseline level, and the age differences in inhibitory strengths are measured indirectly by the magnitude of the *Age × Stimulus condition interaction*. The question of direct interest, however, is how the individual’s responses on the primary task interact with the distractor, and, more importantly, how the strengths of this interaction vary from one age group to another. Thus, the research question is still a matter of interaction, though somewhat more directly identified by use of the predator–prey model. A similar application of the predator–prey model in Equations 3 and 4 to cognitive processes was also suggested by Heath (2000). However, Heath (2000) only used this model as an illustrative example and did not elaborate it in the framework of age-related behavior.

In aging research, Baltes and Baltes (1990) have developed and presented a Selection, Optimization, and Compensation (SOC) model of adaptation across the life span. The SOC model posits that older adults, as a way to compensate for the inevitable decrements in cognitive capacities, opt to prioritize selected goals in life, and they implement strategies to optimize these goals. For instance, during a dual-task experiment in which a group of older and younger adults were asked to perform a cognitive task (memorizing) and a sensorimotor task (walking on a narrow track) concurrently, older adults chose to prioritize walking whereas younger adults chose to prioritize memory performance (Li, Lindenberger, Freund, & Baltes, 2001). This age difference in optimization strategy can be construed as a competitive interaction between two tasks, with one task dominating and forcing the other task to a lower equilibrium level. In the study by Li and colleagues, walking was the dominant task among older adults whereas the memory task clearly dominated among younger adults. Age differences in magnitudes of the intertask interaction terms in Equations 7 and 8 can thus help capture older and younger adults’ differential task prioritization in a dual- or multiple-task context.

The quadratic terms in Equations 7 and 8 were originally incorporated into the predator–prey framework to constrain the growth in the predator and prey populations in the absence of interspecies interactions. In an aging context, these *intratask* interaction components can be conceptualized as part of a learning process—in the absence of other distractors, an individual’s learning is still limited by his or her processing and storage capacity, and eventually it reaches an asymptotic level (i.e., equilibrium) as time progresses. Generally, the values of \( a_{ij} \) and \( a_{ji} \) are negative; this is to impose an upper limit on the growth in task scores. When intertask competitions are dropped, the equilibrium level of a task is defined as a function of the task’s growth rate and its intratask competition rate (Kaplan & Glass, 1995). The resultant trajectory is a sigmoid curve similar in form to the various exponential learning functions examined by Heathcote, Brown, and Mewhort (2000), among others.

The phenomenon of general slowing with age can be represented by the presence and the magnitudes of time delays in Equations 7 and 8. The delay terms \( (\tau_{11}, \tau_{12}, \tau_{21}, \text{and } \tau_{22}) \) can be used to capture the amount of age differences in processing speed. For example, older adults’ failures to exhibit negative priming in some experiments but not others might be due to a delay in processing rather than to deficits in the inhibitory mechanisms per se. For instance, the interaction between a “negative priming” target (i.e., a current target that had appeared previously as a distractor) and a distractor may not fully manifest until some time lag later because of this processing delay. These potential processing delays might induce temporary oscillations in the older adults’ responses, leading to greater “randomness” or “unpredictability” in the older adults’ responses (manifested, e.g., in the form of increased intraindividual variability).

**Objective of this Study**

According to the models explicated herein, the effects of time delay (representing slower processing speed) and greater intratask and intertask interactions (representing decreased inhibitions) are not entirely independent—increased processing delays could theoretically lead to perturbations in an individual’s response patterns, thus producing higher entropy levels.
(Allen et al., 1998a, 1998b) in the individual’s responses. In a similar vein, the intratask and intertask interaction terms can also produce oscillations in the responses when certain parameter ranges are involved. To examine the quantitative and qualitative differences brought by altering these parameters, we performed a simulation by using the model described in Equations 7 and 8. Our goals in presenting these simulations are twofold. First, we wish to illustrate some methodological and theoretical advances in the field of nonlinear dynamical systems. Second, we want to call researchers’ attention to the versatility of differential equation models, as well as several modeling issues that have to be resolved before these models can be adequately evaluated by use of empirical data. Again, our aim in presenting these models is not to question the credibility of testing the associated phenomena separately, but rather to provide an alternative approach for testing their relative impact within the same modeling framework.

**Simulation and Results**

We designed the simulation to generate data as described by Equations 3–8 presented earlier. We performed all simulations using dde23, a MATLAB package developed by Shampine and Thompson (2000). As discussed earlier, intratask competition as represented by the quadratic terms in Equations 3–8 is considered an integrated part of most learning processes. This element is thus retained in all simulations shown here. We started with Equations 3 and 4 and considered the case in which the intertask interaction is of a competitive nature, followed by a case in which two tasks assumed a predator–prey relationship. (In certain parameter ranges, the classical predator–prey relationship as captured by Equations 1 and 2 is known to yield lead-lag cyclic oscillations in the populations of predator and prey. Therefore, these results are not reproduced here. Interested readers can see, e.g., Kaplan & Glass, 1995. Instead, we chose to present results with intratask and intertask interactions starting from Equations 3 and 4.) We then examined the effects of incorporating processing delays by performing simulations based on Equations 7 and 8. To illustrate the applicability of these models, we structured part of our simulations to mimic a complex cognitive task that will be described in the paragraphs that follow. The ranges over which the parameter values were varied are identified in appropriate places as we present the simulation results. To incorporate some individual differences in initial performance levels, we performed all our simulations by using a number of different initial conditions for the two tasks. However, because all these “individuals” conform to the same deterministic model in each simulation (i.e., no disturbance term is involved in the corresponding equation, thus making the dynamics of a system perfectly predictable given information on its past values and the underlying model of change), their trajectories will eventually converge and settle into the same dynamics as time progresses.

In addition to showing our model trajectories in the time domain, we also capture the primary and secondary tasks’ dynamics by using a phase portrait or phase plane (the $x$–$y$ plane is termed the phase plane whereas the entire plot including all the corresponding trajectories is denoted as the phase portrait). A phase portrait captures the dynamics of a construct (or a set of constructs) with respect to its constituent elements. In the current context, a task’s phase portrait reflects changes in an individual’s task scores (or more specifically, the rate of change in the task scores) at different performance levels (i.e., different task scores). Instead of examining changes in task scores over time, we examine, for example, how rapidly an individual’s task scores change when he or she has a low task score versus when the individual is already at a high performance level. We will illustrate the usefulness of a phase portrait with more concrete examples later. Next, we organize our simulation results into three sections to illustrate (a) the dynamics of intertask competition, (b) the intertask interaction of predator–prey nature, and (c) the role of processing delays.

**Intertask Competition and Its Relationship to SOC**

When asked to perform multiple tasks concurrently, older adults sometimes selectively ignore a task to maintain their performance on other tasks (see, e.g., Li et al., 2001; McDowd & Shaw, 2000; Salthouse, Hambrick, Lukas & Dell, 1996). This phenomenon was in fact observed in one of our recent studies (for details see Chow, Nesselroade, & Hamagami, 2003). Essentially, a group of older and younger adults were asked to participate in a multitasking experiment in which they performed a Sternberg memory task, a self-paced arithmetic task, a visual monitoring task, and an auditory discrimination task concurrently by using a computerized program, SYNWORK1 (Elsmore, 1994), that requires extensive use of the computer mouse. For illustrative purposes, here we only present data from two of the four tasks on which age differences in task optimization were most apparent: the visual monitoring task and the arithmetic task. On the arithmetic task (shown in Figure 1B), the participants were asked to give the correct sum of two or three addends at their own pace (depending on the difficulty level of a particular trial) and submit their answer by clicking on the “done” button. On the visual monitoring task (Figure 1C), participants were asked to use the mouse to reset a moving pointer before it reached either end of a horizontal scale. As we varied the difficulty level of the multitasking program (e.g., by speeding up the moving pointer and using more addends for
the arithmetic task), older adults successfully maintained their performance on the visual monitoring task (Figure 2A), but at the expense of the arithmetic task (see Figure 2B). Older adults’ relatively sparse trajectories on the arithmetic task show that they chose to ignore the self-paced arithmetic task and focused instead on the other three tasks, particularly the visual task.

As mentioned earlier, this SOC (Baltes & Baltes, 1990) strategy adopted by older adults can be represented as a competitive interaction between two tasks in which one task dominates over the other task. Simulated data using Equations 3 and 4 with the parameters \( r_1 = 4, r_2 = 3, a_{11} = a_{22} = -0.02, \) and \( a_{12} = a_{21} = -0.03 \) resulted in a situation in which the task with a higher growth rate, denoted here as \( x_1 \), dominated and reached an equilibrium level over time while the other task, \( x_2 \), dropped to zero. (See Figure 3A. All the parameters in the model jointly determine behavioral characteristics of the resultant trajectories, e.g., each task’s respective equilibrium level and the speed with which they settle to these equilibrium levels. For illustration purposes, we chose parameter values that could reflect the dynamic interplay between the two tasks clearly, e.g., to ensure that the two tasks do not settle into their equilibrium levels too quickly or slowly.) Notice that both tasks were assigned the same magnitudes of intertask and intratask competitions, but the task with the higher growth rate dominated.

The convergence in levels of two tasks’ scores with different initial conditions can be seen in the corresponding phase portrait (Figure 3B). The dominant task is represented in the time-domain plot and phase portrait by a solid curve whereas the weaker task was depicted with a dotted curve. In this particular instance, the dominant and weaker tasks were each assigned four initial conditions, with \( x_1(0) = 10, 40, 50, \) and \( 100, \) and \( x_2(0) = 30, 60, 80, \) and \( 100, \) respectively. The rate of change of a task is denoted as \( x'(t) \) in the plots. Regardless of the individual differences in initial scores, the dominant task moved toward its equilibrium level or “fixed point” at \( x_1(t) = 200, \) while \( x_2 \) was forced to zero. An equilibrium point is defined as a point at which a task’s rate of change is equal to zero, that is, where \( x'(t) = 0 \) in the phase portrait.

When the magnitudes of the intertask competitions were reduced to \( a_{12} = a_{21} = -0.003 \) (i.e., the interference imposed from the other task is small relative to the intratask competitions), none of the two tasks demonstrated clear dominance, and both converged toward their respective nonzero equilibrium levels (figures are not shown because of space constraints). This model has qualitative results similar to those of Van Geert’s (1991) coupled logistic model, in which two alternative strategies with sufficiently large growth rates both dominate and stabilize toward some equilibrium levels.

**Intertask Dynamics of Predator–Prey Nature**

We now consider the case in which a primary task and some irrelevant information assume the roles of prey and predator,
respectively. The primary task is interfered with by the growth of irrelevant thoughts, but the latter somehow benefits from this intertask interaction—for example, an individual’s performance on the primary task might “inspire” more task-irrelevant thoughts. We used the same parameters as described in the first simulation, but we lowered the growth rate of the irrelevant thoughts (i.e., $r_2$) from 3.0 to 1.0 (because these thoughts are likely to grow in a smaller magnitude) and changed the sign of its intertask parameter, $a_{21}$, from $-0.03$ to $0.03$, implying that the irrelevant thoughts somehow gained from interacting with the primary task while the latter suffers. The resultant trajectories and phase portrait are shown in Figure 3 (C and D, respectively). The primary task (represented as $x_1$) now settled to a lower equilibrium level, that is, $x_1(t) = 40$, whereas the irrelevant thought continued to benefit from this interaction and attained an equilibrium level of $x_2(t) = 110$.

**Incorporating Processing Delays**

We now present selected results generated with Equations 7 and 8, using the same intertask and intratask parameters as the first simulation ($a_{11} = a_{21} = -0.03$, $a_{12} = a_{22} = -0.02$), but with different magnitudes of time delays. Several plots and their corresponding phase portraits are shown in Figures 4. When the time delays involved were relatively small ($\tau_{11} = \tau_{12} = \tau_{21} = \tau_{22}$ were equal to .4 and .5), the dominant task in the previous simulation, $x_1$, was observed to oscillate in increasing magnitudes, and the other task’s scores still dropped to zero as observed previously (see Figure 4A). As the delay terms were increased to .7, the dominant task was observed to show occasional surges in scores that gave rise to “spikes” of nonzero scores. As the magnitude of delays was increased to 1.0, an individual just failed to maintain a nonzero performance level as a result of these prolonged processing delays (see Figure 4C). However, if the interference from another task was smaller (i.e., the magnitudes of intertask competitions were smaller), this behavioral pattern did not emerge until a longer delay was involved; that is, the processing delays would have to be greater in magnitude before the individual failed to maintain a certain level of optimal scores.

The corresponding phase portraits of the time-delay models provide an alternative representation of the two tasks’ dynamics. When $\tau_{12} = \tau_{21} = \tau_{11} = \tau_{22} = 0.4$, the relatively small delays led to a situation in which $x_2$ still dropped to its equilibrium point at $x_2(t) = 0$, but $x_1$ now oscillated periodically...
in the approximate range of $x_1 = 150$ to 250. The periodic oscillation between these higher and lower scores thus forms a cycle (often termed limit cycle) in the phase portrait (see Figure 4B); instead of having just one equilibrium point, there are now two sets of unstable equilibrium points that fall on the horizontal line defined by $x'(t) = 0$. When the magnitudes of the delay terms were increased to .5, the oscillations in the scores of $x_2$ continued to increase, and this was reflected in the expansion of the limit cycle in the corresponding phase portrait (see Figure 4B). Finally, when the delay terms were increased to .7 and then 1.0, the occasion spikes in scores of $x_1$ formed some sparse limit cycles in the corresponding phase portrait at certain levels of task scores, but mostly clustered around zero otherwise (see Figure 4D).

**DISCUSSION**

As our simulation results demonstrated, it is helpful to represent change in different ways, rather than just along the time domain. We presented a scenario in which intertask interactions of a competitive nature led to outcomes that reflect Baltes and Baltes’ (1990) proposition of SOC. A weaker task can be forced to zero while a dominant task attains a nonzero equilibrium level; this is analogous to a situation in which an older adult selectively directs all of his or her resources to one task while ignoring another task, thus permitting the former task to dominate while the latter drops out.

We also presented a case in which two tasks assumed a predator–prey relation, and the predator—or in our example, the “irrelevant thoughts”—benefited from an interaction with the primary task (i.e., the prey) and stabilized at a higher equilibrium level than the primary task. As a consequence of the detrimental intertask interaction, an individual’s performance on the primary task is greatly compromised. Finally, processing delays were found to induce oscillations in originally stable task scores when the intertask interactions were competitive in nature, but only up to a certain point. If the processing delays exceeded a certain threshold level, the corresponding task scores were observed to drop to zero and remained zero most of the time, with occasional spikes in scores—similar to an individual’s attempt to master a task that is just too difficult for him or her.

The roles played by each parameter in a dynamical model often have to be assessed in relation to the magnitudes of all the parameters. With certain combinations of parameters, different values can at times lead to similar behavioral trajectories. In the present case, this would make it hard to disentangle the effects of delays.
of processing delays from those of intertask competition. Such ambiguity is certainly observed in aging research as well. As suggested by authors such as Salthouse and Meinz (1993), a large portion of the variance in cognitive performance accounted for by measures of interference could also be accounted for by measures of processing speed. Despite limitations of the models we have examined, they allow these two mechanisms to be readily reconciled in a mathematical model.

Even though the models we presented are readily applicable to empirical settings from a theoretical perspective, fitting them to empirical data is not yet a straightforward process. Most of the conventional modeling approaches still yield highly biased parameter estimates when nonlinear models are involved (see, e.g., McArdle & Ghisletta, 2000). Social scientists have long attempted to fit dynamic models in the form of difference or differential equations to empirical data, but unfortunately the quest still faces some unresolved problems even in the simpler case of linear models (see, e.g., Singer, 1990). Nonlinear models add yet another level of complexity (see e.g., Kenny & Judd, 1984; Schumacker & Marcoulides, 1998). However, we subscribe to Molenaar and Raymakers’ (1998) view of the importance of having estimation methods that can fit linear or nonlinear dynamic models directly to empirical data. Currently, the available estimation techniques lag behind novel ideas inspired by dynamical systems theories. Finding more direct estimation techniques that can provide a better linkage between theoretical models and empirical outcomes is therefore a necessary step toward advancing and validating these theories.

These model-fitting issues are, however, well beyond the scope of this article. Admittedly, linear or other general models, such as the confirmatory factor analysis model, may fit many sets of data very well. However, these models tend to be focused on static aspects of the data (e.g., differences in levels of performance), rather than the interesting individual, and thus age differences that reside in the dynamical features (e.g., rates of change).

The typical measurement intervals implemented in psychological studies also pose another problem for estimating parameters from nonlinear dynamical models. For example, the magnitude of processing delay as captured by Equations 7 and 8 can only be estimated if measurements are at very close intervals within a trial, rather than across trials. Furthermore, even if the predicted trajectories appear to resemble the true trajectories very closely, the resulting parameter estimates might still be biased. Therefore, the question we face is not only limited to how much the hypothesized trajectories resemble trajectories from actual data, but also how accurately the true dynamics of a system can be captured. In principle, these matters can be resolved, and the potential payoff makes that resolution worth pursuing energetically.

Finally, besides proposing a model to capture the age differences in cognitive functioning and the qualitative roles played by processing speed and inhibitory mechanisms in quantitative terms, our simulations also highlight the need for having more direct estimation procedures for nonlinear models, and the importance for collecting data within close intervals. Decades of advances in structural equation modeling have provided many tools for examining intraindividual variability and individual differences. It is now time for renewed efforts to elaborate our available tools for modeling the dynamics of behavioral systems.


Shampine, L. F., & Thompson, S. (2000). *Solving DDEs in MATLAB*. Unpublished manuscript, Mathematics Department, Southern Methodist University, Dallas, TX.


Received March 17, 2003
Accepted January 5, 2004
Decision Editor: Margie E. Lachman, PhD